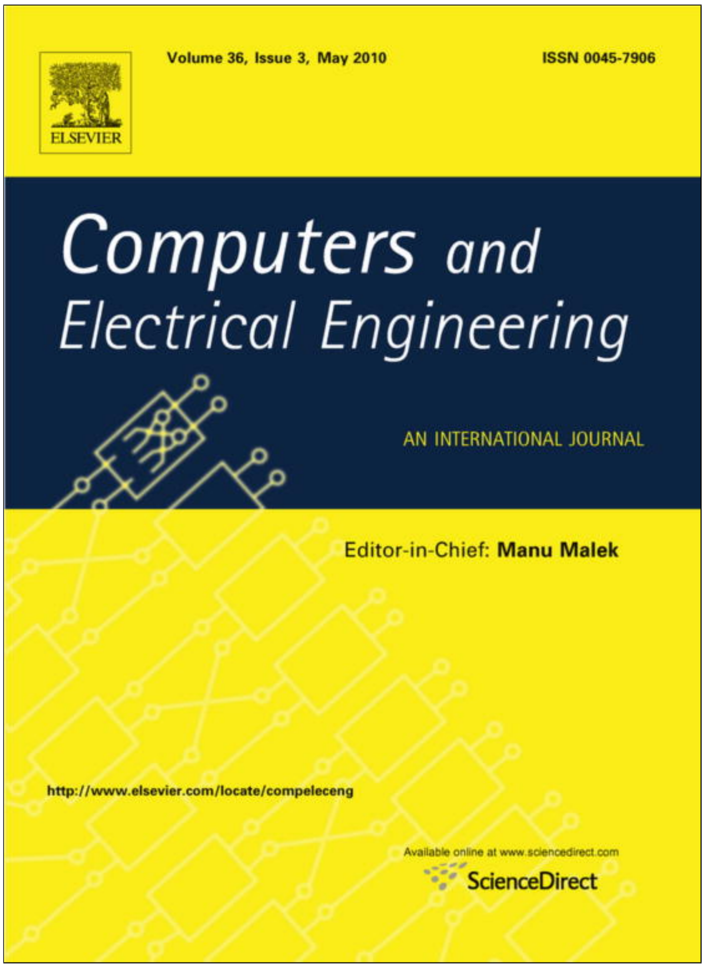
Volume 36, Issue 3, May 2010
ISSN 0045-7906
ELSEVIER
Computers and Electrical Engineering
AN INTERNATIONAL JOURNAL
Editor-in-Chief: Manu Malek
http://www.elsevier.com/locate/compeleceng
Available online at www.sciencedirect.com
ScienceDirect

Computers and Electrical Engineering 36 (2010) 442–454

Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Model and analysis of path compression for mobile Ad Hoc networks

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ARTICLE INFO

Article history:
Received 14 July 2008
Received in revised form 20 August 2009
Accepted 6 November 2009
Available online 24 December 2009

Keywords:
Communication technology
Ad Hoc networks
Path compression
Dynamic model

ABSTRACT

Path compression techniques are efficient on-demand routing optimizing techniques for mobile Ad Hoc networks. However, there is no efficient model for path compression techniques. This paper analyzed the principles and characteristics of path compression algorithms and proposed dynamic model which provided theoretical basis to improve or propose path compression algorithms. This model took the mobility and expansibility of Ad Hoc networks into account and was efficient to analyze or evaluate path compression algorithms. The quantitative relationship and probability expression for pivotal compression events were given based on the model. The simulation results of SHORT (self-healing and optimizing routing techniques) and PCA (path compression algorithm) show that it is a correct and efficient dynamic model for path compression. Finally, some suggestions and application scenarios about the model were proposed.

1. Introduction

Mobile multi-hop Ad Hoc networks are self-creating, self-organizing and self-administrating wireless networks without deploying any kind of infrastructure [1]. These characteristics make MANETs have a bright prospect in military, disaster recovery areas, commerce, education and any other applications without the support of infrastructure. However, the design of routing strategy in MANETs is a challenging work due to the movement, restricted energy and limited processing ability of nodes. The conventional routing protocols which need long-term stability topology and complicated calculation are difficult to be applied to MANETs.

Ad hoc networks need efficient routing protocols which can deal with the dynamic network topology without much overhead. At present, many Ad Hoc routing protocols have been proposed. According to the driven mode, these protocols can be divided into two categories: table-driven routing protocols and on-demand driven routing protocols. Route discovery strategy of table-driven routing protocol is originated from the traditional routing protocol which periodically exchange routing information between nodes, and each node must maintain an updated routing table to other nodes. However, high costs are needed to construct the routing tables with integrated routing information and to maintain the routing information. Therefore, table-driven routing protocols are unfit for those Ad Hoc networks with large scale or low traffic loads. The main idea of on-demand routing protocols is that the process of routing starts only when there are data to be sent. That is to say it is unnecessary to broadcast routing information periodically. Such mechanism releases occupation of network resources effectively. On-demand routing protocols have obvious advantages in scalability and effectiveness. Typical Ad Hoc on-demand routing protocols include AODV [2], DSR [3], TORA [4] etc.

The routing discovery mechanisms (using RREQ/RREP) of typical on-demand routing protocols will establish a shortest (with minimum hop counts) route. However, the mobility of nodes and the absence of global topology information make the path established by routing discovery mechanisms become no longer optimal after a period of time. Even if there are

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doi:10.1016/j.compeleceng.2009.11.002

shorter paths toward destination which are more reasonable and optimal in the current network, routing protocols cannot apperceive them. Routing protocols will retain the original path until the path broken, then the re-routing request initiated. But this is not expected because long paths need more network resources than of short paths, which will lead to poor routing performance. The confirmed disadvantages of long path are: (1) more bandwidth consumption, (2) more energy consumption; (3) increase end-to-end delay; (4) increase the number of routing requests; (5) reduce packet delivery ratio; (6) reduce survival time of routes (increase the probability of destructing the paths); (7) increase the probability of self-conflict phenomenon [5].

Path optimality and stability are important issues for routing protocols when stable paths are more reliable and offer better quality of service (minimal end-to-end delay). Path compression techniques can effectively solve the above problems. In fact, path compression techniques aim at developing a potential shorter path based on on-demand routing protocol. Related work [6–8] has proved that path compression techniques are effective method to reduce the consumption of resources and improve network performance.

In some occasion, path compression techniques try to find the most optimal path to optimize the original path generated by the routing discovery mechanism. Fig. 1a shows a path with eight hop counts (from source *A* to destination *I*) which was generated by routing discovery process. This path may be the shortest path or very close to the shortest path. For the mobility of nodes, this path might be transformed into the shape as shown in Fig. 1b after a period of time: node *J* which not belonged to the original path entered into the transmission range of node *A*, node *E* which belonged to the original path entered into the transmission range of node *J*, node *F* and node *H* entered into the transmission range of each other. Now the ideal path is shown in Fig. 1c, which has only five hop counts. Nevertheless, general routing protocols evaluate the availability of a path by the effectiveness of next hop. As long as the current path is valid, routing information will never be updated (even more optimal path existed). Path compression techniques were proposed to solve this problem.

In general terms, (n,k) short-cuts implies that n routing hops can be reduced to k hops, where $k < n$. When $k < 3$ (it is difficult to deal with the compressions when $k \geqslant 3$), path compressions are possible under two scenarios: (1) two nodes on the active route come close to each other giving rise to a $(n,1)$ short-cuts; (2) a node that is not on the active route comes closer two different nodes on the active route and form a shorter route. This gives rise to $(n,2)$ short-cuts.

In Fig. 2a, the original local path *A*–*B*–*C* has been compressed to *A*–*C*, which is termed as (2,1) short-cut; in Fig. 2b, the original local path *A*–*B*–*C*–*D* has been compressed to *A*–*E*–*D*, which is termed as (3,2) short-cut. If $k > 3$, the compression algorithm will become complicated and lead to instability with extra cost. So the current path compression techniques just consider $(n,1)$ short-cuts and $(n,2)$ short-cuts. In fact, $(n,1)$ short-cuts happened when two nodes which had belonged to the original path but not in one hop count entered into the transmission range of each other. $(n,2)$ short-cuts happened when one node which had not belonged to the original path entered into this path and connected two nodes that located on the original path but more than three hop counts between them. Some path compression algorithms [1,7] were designed according to such property.

This paper aims at the construction of a dynamic and efficient model for path compression techniques. The quantitative relationship and probability expression for pivotal compression events were also given based on the model. The contributions of our compression model are:

(1) make the setting of algorithm parameters more scientific;
(2) provide important reference performance analysis of algorithms;

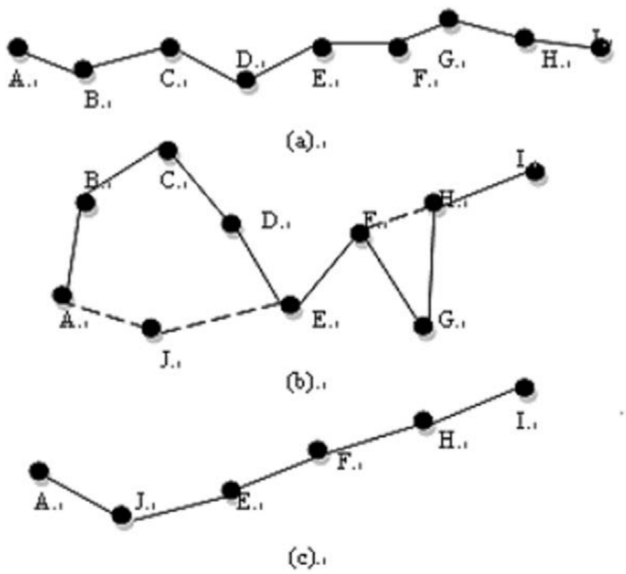

Fig. 1. An example of the path changes.

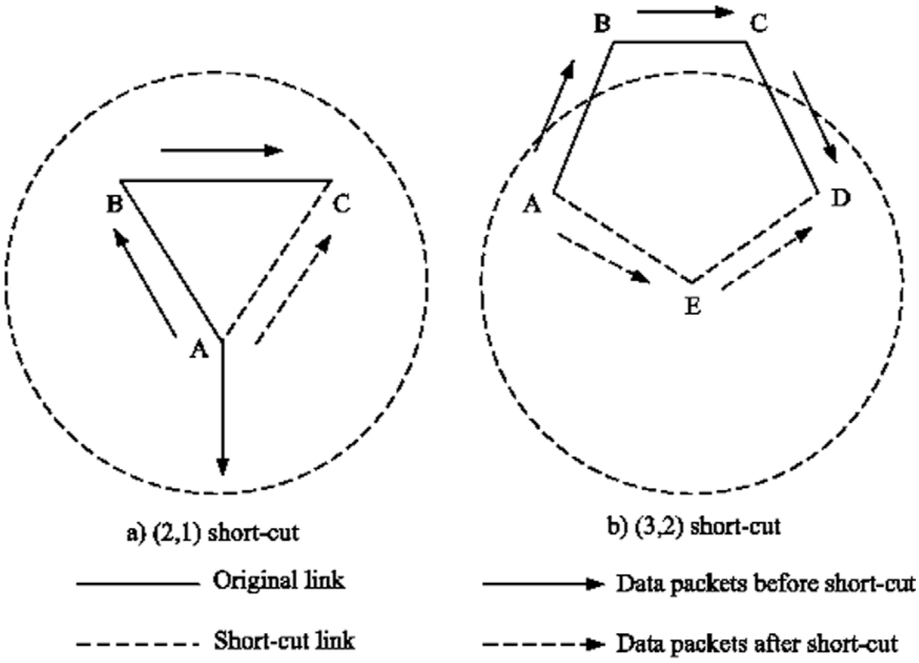

Fig. 2. Basic short-cut formation.

(3) provide theoretical basis for proposition and improvement of path compression algorithms. For examples, we can calculate the average interval of $(n,1)$ short-cuts and $(n,1)$ short-cuts in order to avoid the "vicious" reductions; some path maintenance strategies can be implemented to extend the lifetime of paths when the average interval of $(n,1)$ path break and $(n,2)$ path break can be obtained from our model; in addition, the average interval of ephemeral short-cuts and multiple short-cuts are useful to prevent ephemeral short-cuts and multiple short-cuts periodically.

In next section, we give an overview of the related work concerning path compression techniques. The path compression model and the related probability expression for pivotal compression events were given in Section 3. In Section 4, we evaluate the model and give the detailed results. Finally, Section 5 concludes the paper.

2. Related work

The main idea of path compression techniques is to apperceive optimal paths and compress the original paths through adjusting the routing table of corresponding nodes. An important principle of designation is to optimize the path with reasonable increase of costs.

ARS (Automatic Route Shortening) in DSR protocol [3] is one of the first path compression algorithms for source routing. In ARS, each node maintains a gratuitous route reply table which contains local address, predecessor address and lifetime for each item. ARS is built on source routing, thus the routing update packets must be delivered to the source node, and the compression cost is high. In addition, only $(n,1)$ short-cuts are exploited in ARS.

Gui proposed SHORT [7,9] whicah is a distributed path compression algorithm. By utilizing the promiscuous mode of operation, nodes in the network monitor data packets to exploit $(n,1)$ short-cuts and $(n,2)$ short-cuts as and when feasible. SHORT utilizes "*stable period*" to hold back the multiple short-cuts in order to make the paths steady. Compared to AODV and DSR, SHORT has significant improvement at the performance of delivery rate and routing overhead. However, SHORT will be invalid in following cases: (1) $(2,1)$ short-cuts containing the source node; (2) any reduction containing the destination node.

Jiao proposed NAOR (Neighbor-aware Optimizing Routing) [10] which can implement $(2,1)$ short-cuts containing the source node. PCA [8] improved SHORT in the following areas: (1) PCA extends the reduction to the paths containing multiple sources or destinations. Zig-zag paths are also feasible in PCA; (2) PCA reduces the overhead since only $(n,2)$ short-cuts need deliver routing update packets; (3) PCA distinguishes the paths with broadcast sequence number in order to avoid compression conflict on different paths. PCA outperforms AODV and SHORT in terms of average routing hop count, end-to-end delay and routing overhead. To hold back ephemeral short-cuts, PCA reverted back to the original route using auxiliary route tables when ephemeral short-cuts happened. But such mechanism cannot avoid the ephemeral short-cuts fundamentally, and the auxiliary routing table requires extra space in each node.

DPS (Dynamic Path Shortening) [6] is a proximity-based dynamic path shortening algorithm which apperceives the compressions by handshake mechanism. DPS uses Smoothed Signal-to-Noise Ratio (SSNR) to indicate the location of nodes and link quality. This strategy is useful to guarantee the link quality although only $(n,1)$ short-cuts is availability in DPS. However, it is expensive to compress since three extra control packets are necessary for each compression.

Table 1
The comparison of typical path compression techniques of Ad Hoc networks.

Compression techniques	ARS	DPS	SHORT	PCA
Type of routing protocol	On-demand source routing	On-demand distributed routing	On-demand distributed routing	On-demand distributed routing
Perceiving short-cut	Interception	Smoothing SNR	Interception	Interception
Routing update	Direct update	Handshake	Direct update	Direct update
Extra storage space	Yes	No	Yes	Yes
Number of control message	Few	Many	Middle	Few
Supported short-cut	$(n,1)$	$(n,1)$	$(n,1)$ and $(n,2)$	$(n,1)$ and $(n,2)$
Model of short-cut	No	No	Simple static model	No
Multiple short-cuts	Automatic route shortening table	No solution	Routing stable period	No solution
Ephemeral short-cuts	No solution	No solution	No solution	Auxiliary route table
Short-cut contain source or destination	Support	Support	Not support	Support
Zig-zag paths	Not support	Not support	Not support	Support
Bidirectional short-cut	Not support	Not support	Not support	Partly support
Link quality	Good	Good	No guarantee	No guarantee
Efficiency of short-cut	No guarantee	No guarantee	Good	Good

We had summarized the typical path compression techniques in [11], and Table 1 shows the characteristics and application scenarios, respectively. We can see that there are some shortcomings in current path compression algorithms, for examples, ephemeral short-cut problem and multiple short-cuts problem are not solved fundamentally; some parameters such as "*stable period*" can not be evaluated scientifically. An efficient compression model is needed urgently to provide the theoretical basis for proposition and improvement of compression algorithms. Despite a static model which assumes that each node is immobile and the link is connected at any time was proposed in [7], it is usually unrealistic for mobile Ad Hoc network environment.

3. Dynamic model of path compression for Ad Hoc networks

In this Section, the dynamic model was described by three parts: (1) dynamic model of $(n,1)$ short-cuts; (2) dynamic model of $(n,2)$ short-cuts; (3) computation and analyses of compression probability for pivotal compression events.

3.1. Dynamic model of (n, 1) short-cuts

The $(n,1)$ short-cuts only happened when two nodes on the active route come closer to each other. So, we established the $(n,1)$ short-cuts model on the basis of following assumptions:

(1) All nodes were randomly distributed in the area S and the total number of nodes in the network is denoted as N.
(2) All nodes moved according to Random Direction mobility model in period t.
(3) Each node has the same maximum transmission range, denoted as r.

Based on the above assumptions, we can calculate the probability of a new node v entering into the transmission range of node u and the probability of an existing neighbor node v leaving the transmission range of node u. Here we denote the effective transmission region of node u as $D\ (u,r)$, that means the area of a circle with it's center as u, radius as r. The relative distance between node v and node u is BC, where $BC = d = v_0^* t$ (v_0 is the relative speed which can be calculated in node v).

3.1.1. The probability of node v entering the region D(u, r)

We denote $P_{in(n,1)}$ as the probability of node v entering $D(u, r)$ during time interval t. $P_{in(n,1)}$ can be calculated under following two situations:

Situation 1: $0 < d < 2r$. As is shown in Fig. 3, node u locates in the position A, node v is in the position B. the maximum transmission range of nodes is $AC = r$. In Fig. 3, assuming the area of the shadow region is S_1 and the probability is $P_{in(n,1)\_1}$ in this situation. Since $0 < d < 2r$, $x \in (r, \ r + d]$ is necessary in order to ensure that node v can enter the region $D(u,r)$ after moving distance d. Assume that the distance between node u and v is $AB = x > r$), and the probability is $\frac{2\pi x}{S}$ since nodes can located in the area S according to uniformity distribution. For any moving direction within $[0, 2\pi)$, the probability of node v entering the region $D(u, r)$ is $\frac{S_1}{\pi d^2}$ under above conditions. Putting it altogether we get that:

$$P_{in(n,1)} = P_{in(n,1)\_1} = \int_r^{r+d} \frac{2\pi x}{S} \quad \frac{S_1}{\pi d^2} dx = \int_r^{r+d} \frac{2xS_1}{Sd^2} dx \tag{1}$$

where $S_1 = \alpha_1 r^2 + \alpha_2 d^2 - \sin\alpha_1 \sin\alpha_2 (r^2 + d^2)$; $\alpha_1 = \angle CAB = \arccos\frac{x^2+r^2-d^2}{2xr}$; $\alpha_2 = \angle CBA = \arccos\frac{x^2+d^2-r^2}{2xd}$

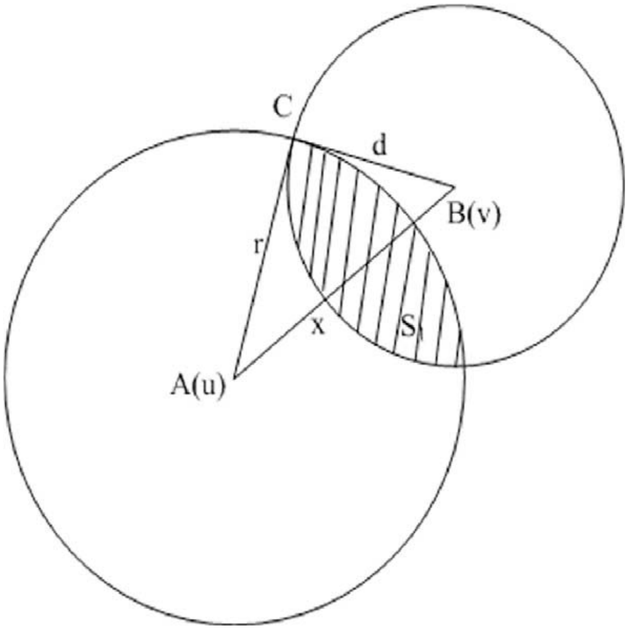

Fig. 3. Calculation for $P_{in(n,1)\_1}$ and $P_{in(n,1)\_21}$.

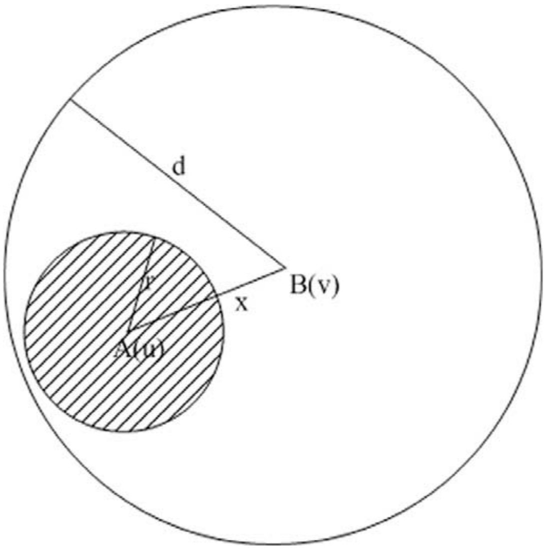

Fig. 4. Calculation for $P_{in(n,1)\_22}$.

Situation 2: $d \geqslant 2r$. According to the bounds of x, $P_{in(n,1)}$ is sum of $P_{in(n,1)\_21}$ and $P_{in(n,1)\_22}$. $P_{in(n,1)\_21}$ is the probability of node v entering the region $D(u,r)$ within time t in the case illustrated by Fig. 3, and $x \in (r,\ r+d]$ is necessary in this case. $P_{in(n,1)\_22}$ is the probability of node v entering the region $D(u,r)$ within time t in the case illustrated by Fig. 4. In this case, $x \in (r,\ d-r]$ is necessary. Then:

$$P_{in(n,1)} = P_{in(n,1)\_21} + P_{in(n,1)\_22} = \int_r^{r+d} \frac{2\pi x}{S}\frac{S_1}{\pi d^2}dx + \int_r^{d-r}\frac{2\pi x}{S}\frac{\pi r^2}{\pi d^2}dx = \int_{d-r}^{d+r}\frac{2xS_1}{Sd^2}dx + \frac{\pi r^2(d-2r)}{Sd} \quad (2)$$

3.1.2. The probability of node v leaving the region D(u, r)

We assume the probability of node v leaving $D(u,r)$ during time interval t as $P_{out(n,1)}$, which can be calculated under following three situations:

Situation 1: $0 < d < r$. As shown in Fig. 5, we assume the area of shadow region as S_2 and the probability of node v leaving the region $D(u,r)$ within time t as $P_{out(n,1)\_1}$. $x \in (r,\ r-d]$ is necessary in this situation, then:

$$P_{out(n,1)} = P_{out(n,1)\_1} = \int_{r-d}^{r}\frac{2\pi x}{S}\frac{S_2}{\pi d^2}dx = \int_{r-d}^{r}\frac{2xS_2}{Sd^2}dx \quad (3)$$

where $S_2 = (\pi-\alpha_2)d^2 - (\alpha_1 r^2 - xd\sin\alpha_2)$; $\alpha_1 = \angle CAB = \arccos\frac{x^2+r^2-d^2}{2xr}$; $\alpha_2 = \angle CBA = \arccos\frac{x^2+d^2-r^2}{2xd}$.

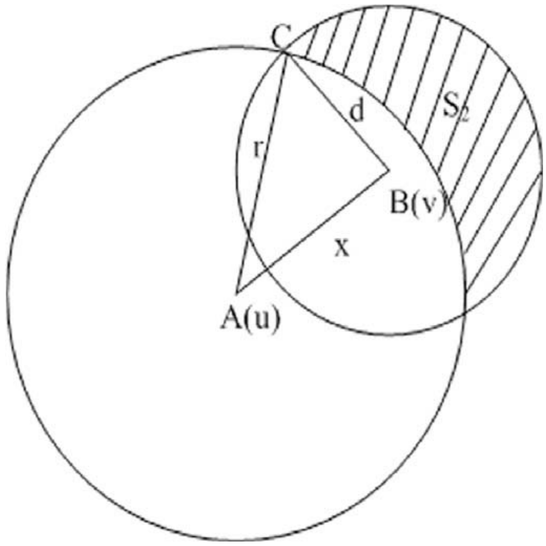

Fig. 5. Calculation for $P_{out(n,1)\_1}$ and $P_{out(n,1)\_21}$.

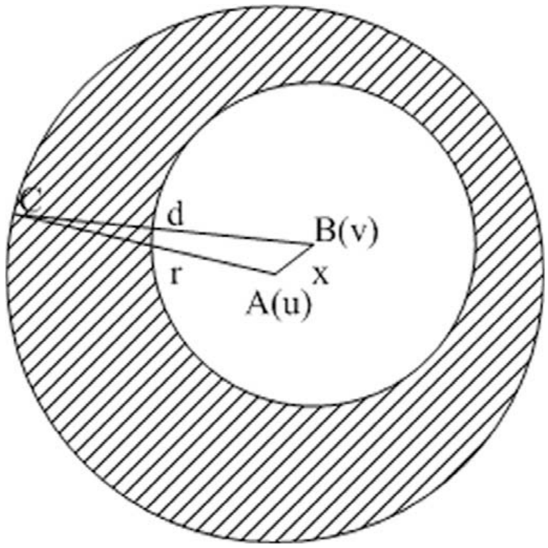

Fig. 6. Calculation for $P_{out(n,1)\_22}$ and $P_{out(n,1)\_3}$.

Situation 2: $r \leqslant d < 2r$. According to the bounds of x, the value of $P_{out(n,1)}$ is the sum of the value of $P_{out(n,1)\_21}$ and $P_{out(n,1)\_22}$. $P_{out(n,1)\_21}$ is the probability of node v leaving the region $D(u, r)$ within time t in the case illustrated by Fig. 5, and $x \in (d-r, r]$ is necessary in this case. $P_{in(n,1)\_22}$ is the probability of node v entering the region $D(u, r)$ within time t in the case illustrated by Fig. 6. In this case, $x \in (0, \; d-r]$ is necessary. Then:

$$P_{out(n,1)} = P_{out(n,1)\_21} + P_{out(n,1)\_22} = \int_{d-r}^{r} \frac{2\pi x}{S} \frac{S_2}{\pi d^2} dx + \int_{0}^{d-r} \frac{2\pi x}{S} \frac{\pi(d^2 - r^2)}{\pi d^2} dx = \int_{d-r}^{r} \frac{2xS_2}{Sd^2} dx + \frac{\pi(d+r)}{Sd^2}(d-r)^3 \tag{4}$$

Situation 3: $d \geqslant 2r$. This situation can be illustrated by Fig. 6 with $x \in (0, r]$. We denote $P_{out(n,1)\_3}$ as the probability of node v leaving the region $D(u, r)$ within time t, then:

$$P_{out(n,1)} = P_{out(n,1)\_3} = \int_{0}^{r} \frac{2\pi x}{S} \frac{\pi(d^2 - r^2)}{\pi d^2} dx = \frac{\pi(d^2 - r^2)r^2}{Sd^2} \tag{5}$$

3.2. Dynamic model of (n,2) short-cuts

The $(n,2)$ short-cuts are caused by a node which did not belong to the original path entered into the public transmission region of two nodes that located on the original path. We established the $(n,2)$ short-cuts model on the basis of following assumptions:

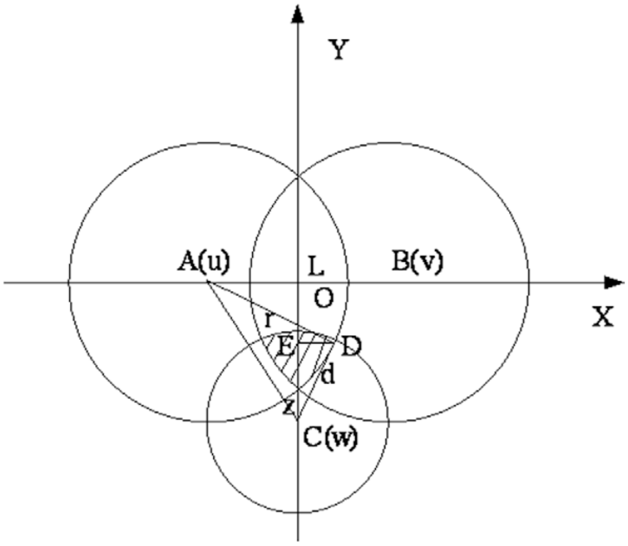

Fig. 7. Calculation for $P_{in(n,2)\_1}$ and $P_{in(n,2)\_21}$.

(1) All nodes were randomly distributed in the area S and the total number of nodes in the network is denoted as N.
(2) All nodes moved according to Random Direction mobility model in period t.
(3) Each node had the same maximum transmission range, denoted as r.
(4) We took a typical case in $(n,2)$short-cuts into account: If the $(n,2)$ short-cut between node u and v was caused by node w, node w moves on the perpendicular line between node u and node v. This assumption is reasonable since we aimed at simplifying the model in Ad Hoc network environment. It is unrealistic to get more parameters by extra control packets in resource restricted Ad Hoc networks. So we paid more attention to the usability and practicability of the model in such environment. The simulations in Section 4 showed that our model was correct and efficient. It meant that our model can be applied without assumption (4).

Based on the above assumptions, we can calculate the probability of node w entering into the public transmission region of node u and$v$ and the probability of an existing neighbor node w leaving the public transmission region of node u and v. Here we denote the public transmission region of node u and v as $D(u,v,r)$ which is the overlap generated by two circles with radius r, center u and v respectively. We denote random variables L which is uniformity distributed in $(r, 2r]$ as the distance between node u and node v. We established the coordinate system with the line connected node u and node v as x-axis and the perpendicular line between node u and node v as y-axis (see Fig. 7). The relative movement distance within time t from node w to the coordinate center is d, where $d = v_0^* t$.

3.2.1. The probability of node w entering the region D(u, v, r)

We assume the probability of node w accessing $D(u,v,r)$ during time interval t is $P_{in(n,2)}$, which can be calculated under following two situations:

Situation 1: $0 < d < \sqrt{4r^2 - L^2}$. As is shown in Fig. 7, node u is in the position A, node v in B and node w in C, the maximum transmission distance of nodes is $AB = r$, the distance between node w and coordinate origin O is $CO = z\left(z > \frac{\sqrt{4r^2-L^2}}{2}\right)$. Since $0 < d < \sqrt{4r^2 - L^2}, z \in \left(\frac{\sqrt{4r^2-L^2}}{2}, \frac{\sqrt{4r^2-L^2}}{2} + d\right]$ is necessary to ensure that node v can enter $D(u,v,r)$. *In* Fig. 7, assuming the shadow area is S_3, and the probability is denoted as $P_{in(n,2)\_1}$ in this situation, then:

$$P_{in(n,2)} = P_{in(n,2)\_1} = \int_{\frac{\sqrt{4r^2-L^2}}{2}}^{\frac{\sqrt{4r^2-L^2}}{2}+d} \left(\frac{2\pi z}{S} \cdot \frac{S_3}{\pi d^2}\right) dz \tag{6}$$

The deduction is similar with those for formula (1). The equations of disk A, B, C are: $\left(x + \frac{L}{2}\right)^2 + y^2 = r^2, \left(x - \frac{L}{2}\right)^2 + y^2 = r^2, (x)^2 + (y+z)^2 = d^2$, then: $S_3 = 2\int_0^{|ED|} \sqrt{r^2 - \left(x + \frac{L}{2}\right)^2} - \left(\sqrt{d^2 - x^2} - z\right) dx, ED = CD\sin(\alpha_1 - \alpha_2) = d\sin(\alpha_1 - \alpha_2)$, where, $\alpha_1 = \angle ACD = \arccos\frac{\left(\frac{L}{2}\right)^2 + z^2 + d^2 - r^2}{2d\sqrt{\left(\frac{L}{2}\right)^2 + z^2}} = \arccos\frac{L^2 + 4z^2 + 4d^2 - 4r^2}{d\sqrt{L^2 + 4z^2}}; \alpha_2 = \angle ACO = \mathrm{arctg}\frac{L/2}{z} = \mathrm{arctg}\frac{L}{2z}$.

Situation 2: $d \geqslant \sqrt{4r^2 - L^2}$. According to the range of z, $P_{in(n,2)}$ is the sum of $P_{in(n,2)\_21}$ and $P_{in(n,2)\_22}$, $P_{in(n,2)\_21}$ is shown in Fig. 7 with $z \in \left(\frac{\sqrt{4r^2-L^2}}{2}, \frac{\sqrt{4r^2-L^2}}{2} + d\right], P_{in(n,2)\_22}$ is shown in Fig. 8 with $z \in \left(\frac{\sqrt{4r^2-L^2}}{2}, d - \frac{\sqrt{4r^2-L^2}}{2}\right]$ (assume that the shadow area is denoted as S_4 in Fig. 8), then:

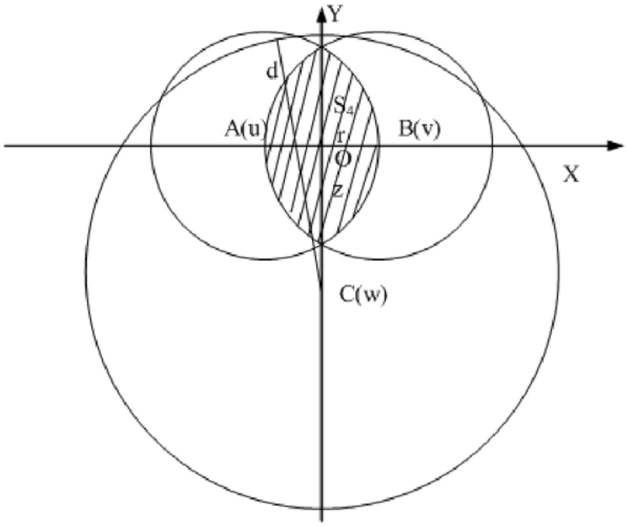

Fig. 8. Calculation for $P_{pin(n,2)\_22}$.

$$P_{in(n,2)} = P_{in(n,2)\_21} + P_{in(n,2)\_22} = \int_{\frac{\sqrt{4r^2-L^2}}{2}}^{d+\frac{\sqrt{4r^2-L^2}}{2}} \frac{2\pi z}{S}\frac{S_3}{\pi d^2}dz + \int_{\frac{\sqrt{4r^2-L^2}}{2}}^{d-\frac{\sqrt{4r^2-L^2}}{2}} \frac{2\pi z}{S}\frac{S_4}{\pi d^2}dz \quad (7)$$

where $S_4 = 2\left(\pi r^2 \frac{2\arccos\frac{L}{2r}}{2\pi} - \frac{L}{2}\sqrt{r^2-\left(\frac{L}{2}\right)^2}\right) = 2r^2\arccos\frac{L}{2r} - \frac{L}{2}\sqrt{4r^2-L^2}$

3.2.2. The probability of node w leaving the region D(u, v, r)

We assume the probability of node w leaving $D(u,v,r)$ during time interval t is $P_{out(n,2)}$ which can be calculated under three situations:

Situation 1: $0 < d < \frac{\sqrt{4r^2-L^2}}{2}$. As is shown in Fig. 9, $z \in \left(\frac{\sqrt{4r^2-L^2}}{2} - d, \frac{\sqrt{4r^2-L^2}}{2}\right]$ is necessary to ensure that node v can leave $D(u,v,r)$. Assume that the area of shadow region is S_5 and the probability is $P_{out(n,2)\_1}$ in this situation, then we get:

$$P_{out(n,2)} = P_{out(n,2)\_1} = \int_{\frac{\sqrt{4r^2-L^2}}{2}-d}^{\frac{\sqrt{4r^2-L^2}}{2}} \frac{2\pi z}{S}\frac{S_5}{\pi d^2}dz = \int_{\frac{\sqrt{4r^2-L^2}}{2}-d}^{\frac{\sqrt{4r^2-L^2}}{2}} \frac{2zS_5}{Sd^2}dz \quad (8)$$

where $S_5 = \pi d^2 - 2\int_0^{ED}\left(\sqrt{r^2-x^2} - z\right) + \sqrt{r^2-\left(x+\frac{L}{2}\right)^2}dx$; $ED = CD\sin(\alpha_1-\alpha_2) = d\sin(\alpha_1-\alpha_2)$; The deductions for α_1 and α_2 can be found in Section 3.2.1.

Situation 2: $\frac{\sqrt{4r^2-L^2}}{2} \leqslant d < \sqrt{4r^2-L^2}$. According to the range of z, $P_{out(n,2)}$ is the sum of $P_{out(n,2)\_21}$ and $P_{out(n,2)\_22}$. The calculation for $P_{out(n,2)\_21}$ is shown in Fig. 9 with $z \in \left(d - \frac{\sqrt{4r^2-L^2}}{2}, \frac{\sqrt{4r^2-L^2}}{2}\right]$. The calculation for $P_{out(n,1)\_22}$ is shown in Fig. 10 with $z \in \left(0, d - \frac{\sqrt{4r^2-L^2}}{2}\right]$. Assume that the area of shadow region is S_6 in Fig. 10, we can get:

$$P_{out(n,2)} = P_{out(n,2)\_21} + P_{out(n,2)\_22} = \int_{d-\frac{\sqrt{4r^2-L^2}}{2}}^{\frac{\sqrt{4r^2-L^2}}{2}} \frac{2\pi z}{S}\frac{S_5}{\pi d^2}dz + \int_0^{d-\frac{\sqrt{4r^2-L^2}}{2}} \frac{2\pi z}{S}\frac{S_6}{\pi d^2}dz \quad (9)$$

where $S_6 = \pi d^2 - 2\left(\frac{\arccos\frac{L}{2r}}{2\pi}\pi r^2 - \frac{L}{2}\sqrt{r^2-\left(\frac{L}{2}\right)^2}\right)$

Situation 3: $d \geqslant \sqrt{4r^2-L^2}$. This situation can be illustrated by Fig. 10 with $x \in (0, r]$., we assume that the probability is $P_{out(n,2)\_2}$, then:

$$P_{out(n,2)} = P_{out(n,2)\_2} = \int_0^{\frac{\sqrt{3}}{2}r} \frac{2\pi z}{S}\frac{S_6}{\pi d^2}dz \quad (10)$$

3.3. Calculation and analysis of compression probability based on dynamic model

Based on above dynamic model of path compression, the following conclusions about the probability of pivotal compression events during time interval t can be made (excluding the short-cuts containing sources or destinations):

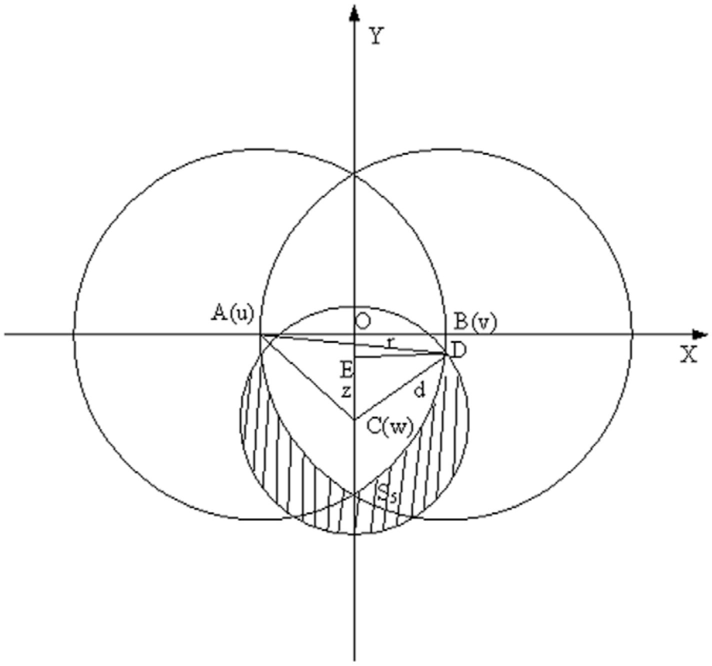

Fig. 9. Calculation for $P_{out(n,2)\_1}$ and $P_{out(n,2)\_21}$.

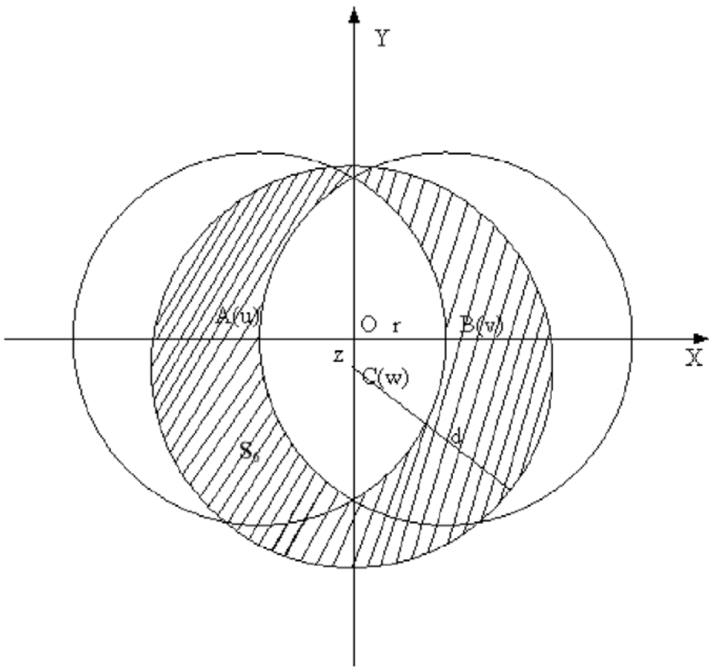

Fig. 10. Calculation for $P_{out(n,2)\_22}$ and $P_{out(n,2)\_3}$.

Conclusion 1: Assuming a network with N nodes, for given node u on the path with $M(N \geqslant M > 3)$ nodes, the probability of $(n,1)$ short-cuts on node u is:

$$p_{(n,1)\_join} = 1 - (1 - p_{in(n,1)})^{M-3} \tag{11}$$

Conclusion 2: The probability of path break at node u is (under the condition that had occurred $(n,1)$ short-cuts ago at node u):

$$p_{(n,1)\_leave} = p_{out(n,1)} \tag{12}$$

Conclusion 3: Assuming a network with N nodes, for given node u on the path with $M(N \geqslant M > 3)$ nodes, the probability of $(n,2)$ short-cuts on node u is:

$$p_{(n,2)\_join} = 1 - (1 - p_{in(n,2)})^{N-M} \tag{13}$$

Conclusion 4: The probability of path break at node u is (under the condition that had occurred $(n,2)$ short-cuts ago at node u):

$$p_{(n,2)\_leave} = p_{out(n,2)} \tag{14}$$

Conclusion 5: Assuming a network with N nodes, for given node u on the path with $M(N-1 > M > 3)$ nodes, the probability of multiple short-cuts on node u is:

$$p_{ms} = 1 - (1 - p_{in(n,2)})^{N-M} - p_{in(n,2)}(1 - p_{in(n,2)})^{N-M-1} \tag{15}$$

Conclusion 6: Assuming a network with N nodes, for given node u on the path with $M(N \geqslant M > 3)$ nodes, the occurrence probability of ephemeral short-cuts on node u is:

$$p_{es} = p_{(n,2)\_join} \cdot p_{(n,2)\_leave} = (1 - (1 - p_{in(n,2)})^{N-M}) \cdot p_{out(n,2)} \tag{16}$$

We implemented the model on MATLAB and got the theoretic probability with given parameters. Due to space limitations, we only made an example of $(n,2)$ short-cuts here. Assuming $S = 4^{*}10^{6}$, $r = 625$, $M = 8$, and according to conclusion 3, we analyzed speed v_0 and the number of nodes N under the ideal conditions. Fig. 11 shows the relationship between the time interval and speed of nodes under different compression probability with 50 nodes. According to our analysis, under the same compression probability and network scale, the time interval for $(n,2)$ short-cut events is inversely proportional to speed. It means the faster the speed is, the shorter the time interval of $(n,2)$ short-cuts will be, namely the occurrence of $(n,2)$ short-cuts are easier. Fig. 12 shows the relationship between time interval and network scale under different compression probability with speed 5 m/s. Under the same compression probability and speed, the larger the network scale is, the shorter the time interval of $(n,2)$ short-cuts will be.

4. Simulation results

This paper implemented SHORT and PCA in JiST/SWANS [12] to verify the correctness and effectivity of dynamic model. In the simulations, all nodes are uniformly distributed in 800 × 800 m region. The transmission radius is 225 m, and 802.11 DCF is used as the MAC layer protocol. Two particular nodes were selected as the source and destination node on the active path. The default packet size is 500 bytes. The source node creates one packet each second. It is easy to apperceive short-cuts with high traffic. We used AODV as on-demand routing protocol. All nodes move according to Random Direction mobility model. The simulation time is 36,000 s.

We have made statistical analysis for the number of $(n,1)$ short-cuts, $(n,1)$ path breaks, $(n,2)$ short-cuts, $(n,2)$ path breaks, multiple short-cuts and ephemeral short-cuts of SHORT and PCA with different speed and network scale. Then we compared

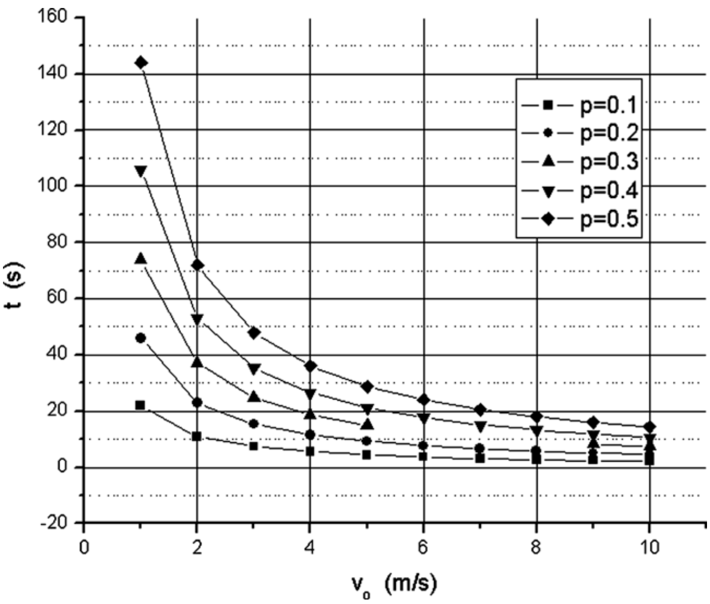

Fig. 11. The period versus the speed with respect to different values of $p_{(n,2)\_join}$.

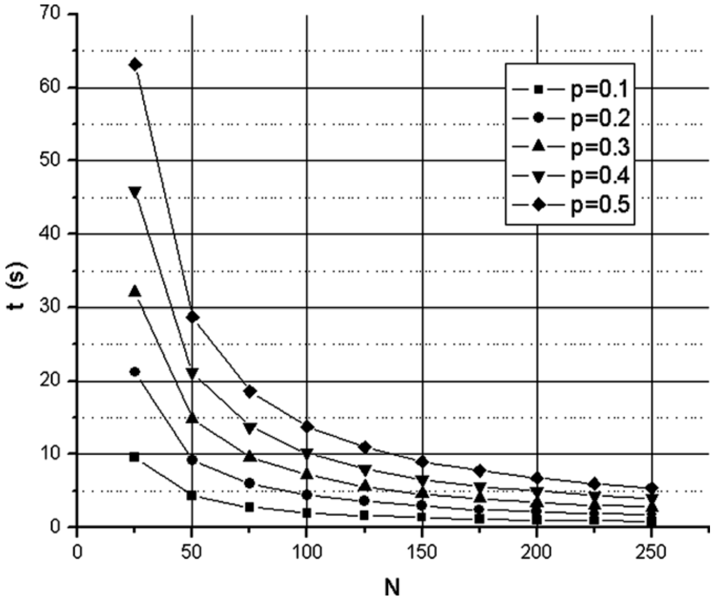

Fig. 12. The period versus NW size with respect to different values of $p_{(n,2)\_join}$.

the statistical data to our theoretical data derived from the model. To collect above statistical data, we add Compression Record Table (CRT) in SHORT and PCA. CRT records local address, next hop address of compression, destination node address, compression time stamp and compression type ($(n,1)$ short-cut or $(n,2)$ short-cut) of each compression. When the local route table was modified, and there were the same local addresses and destination node addresses but different next hop addresses of compression in CRT, then the compression was regarded as broken. We can distinguish the path break as $(n,1)$ break or $(n,2)$ break through the type of compression. If a path break occurred at the same node after the $(n,2)$ short-cuts within three seconds, it would be regarded as Ephemeral short-cuts If a given node received $(n,2)$ short-cuts requests two times within 2 ms, it would be regarded as multiple short-cuts. Note that all statistical data are accumulative data, that is to say the same short-cuts at different timestamp are regarded as different short-cuts.

Table 2 shows the simulation data and theoretical data of SHORT with 100 node network under different speed. Table 3 shows the similar comparisons with speed of 30 m/s under different network scale. All simulation data are the average values of fifty simulations. We got the average interval of compression events from the probability equations introduced in Section 3.3 since the number of short-cuts conform to binomial distribution (let $L = r = 225$, M is determined on the statistic results of simulations). Then, it is easy to get the quantity of these compression events since the simulation time was known. In addition, we define average error $\lambda = \frac{|\text{theoretic value-average statistical value}|}{\text{average statistical value}} \times 100\%$. We can see from Table 2 that statistical data of each testing items is trend to increase with increased network mobility. It conforms to the corresponding conclusion of dynamic compression model proposed in Section 3.3. In addition, the statistical data and theoretic data were very close, and the average error is acceptable. This indicates that our model is correct and effective. Besides, we can see from Table 3 that $(n,1)$ short-cuts, $(n,1)$ breaks, $(n,2)$ breaks are independent on network scale. This conclusion can also be made from the

Table 2
The comparison of statistical value and our theoretic value for SHORT with different speed.

Speed (m/s)		10	15	20	25	30	35	40	45	50	λ (%)
The number of $(n,1)$ short-cuts	Statistical	665.2	885.4	1717.2	1736.2	2372.5	2811.2	2912.4	3649.3	3983.5	10.14
	Theoretic	720	1080	1440	1800	2160	2520	2880	3240	3600	
The number of $(n,1)$ breaks	Statistical	1.8	2.8	3.8	4.9	5.8	6.9	6.9	8.3	9.2	4.62
	Theoretic	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9	
The number of $(n,2)$ short-cuts	Statistical	1780	2826	4080	4083	5560	6564	7582	8991	9529	7.37
	Theoretic	1800	2700	3600	4500	5400	6300	7200	8100	9000	
The number of $(n,2)$ breaks	Statistical	3.8	5.1	7.5	8.7	10.1	12.1	12.3	14.6	19.8	7.41
	Theoretic	3.6	5.4	7.2	9	10.8	12.6	14.4	16.2	18	
The number of multiple short-cuts	Statistical	625	972	1084	1495	2118	2213	2915	3600	3619	10.59
	Theoretic	692	1038	1385	1731	2077	2423	2769	3115	3462	
The number of ephemeral short-cuts	Statistical	353	425	727	880	934	1232	1338	1541	1542	7.41
	Theoretic	325.5	488.2	651	813.7	976.5	1139.2	1302	1464.7	1627.5	

Table 3
The comparison of statistical value and our theoretic value for SHORT with different network scale.

The number of nodes		50	100	150	200	250	300	350	400	λ (%)
The number of $(n,1)$ short-cuts	Statistical	2154.4	2783.8	2699.2	2122	2843.5	2380.8	2163.8	2205.2	10
	Theoretic	2160	2160	2160	2160	2160	2160	2160	2160	
The number of $(n,1)$ breaks	Statistical	5.8	5.4	4.3	5.4	5.3	5.1	6.0	5.8	4.28
	Theoretic	5.4	5.4	5.4	5.4	5.4	5.4	5.4	5.4	
The number of $(n,2)$ short-cuts	Statistical	1923.6	5145	6768.8	7989.2	9568.5	11000.6	14200.4	14400.8	10.01
	Theoretic	1964	5400	7200	8307.7	9818.2	13,500	15428.6	18,540	
The number of $(n,2)$ breaks	Statistical	10.6	12	12.6	12.7	11.4	13.8	10.2	10.4	9.74
	Theoretic	10.8	10.8	10.8	10.8	10.8	10.8	10.8	10.8	
The number of multiple short-cuts	Statistical	1729.5	1876.2	2019.7	3461.1	5519.6	7021.8	7690.2	8582.3	7.58
	Theoretic	1878.3	2077	2250	3600	5142.9	6352.9	7200	8307.7	
The number of ephemeral short-cuts	Statistical	511.5	892.2	1311.2	1835.2	2499.8	2576.2	3076.4	3709.6	7.05
	Theoretic	586.6	976.5	1235.4	1857.1	2387.5	2846.8	3268.9	3857.1	

Table 4
The comparison of statistical value and our theoretic value for PCA with different speed.

Speed (m/s)	10	15	20	25	30	35	40	45	50	λ (%)
The number of $(n,1)$ short-cuts	617.2	1033.2	1629.2	1636.2	2252.7	2791.6	2942.1	2972.8	3991.3	8.61
The number of $(n,1)$ breaks	1.8	2.4	3.4	4.4	5.6	6.4	6.8	7.6	9.8	5.18
The number of $(n,2)$ short-cuts	1750	2553	3460.8	4169	5192.4	6207	7222.8	8232.6	9287.6	3.46
The number of $(n,2)$ breaks	4	5.2	7.5	8.5	10.6	13.1	16.4	16.8	18.6	5.38
The number of multiple short-cuts	675	1189	1217	1504	2115.7	2475.8	2917	3218.6	3608.1	5.17
The number of ephemeral short-cuts	311.9	413.5	694.2	824.8	936.4	1107.2	1282.8	1488.4	1590.2	4.73

Table 5
The comparison of statistical value and our theoretic value for PCA with different network scale.

The number of nodes	50	100	150	200	250	300	350	400	λ (%)
The number of $(n,1)$ short-cuts	2145.2	2738.6	2698.2	2290.8	2436.4	2605	2731.4	2082.3	12.5
The number of $(n,1)$ breaks	5.4	5.2	4.6	5.6	5.4	6.6	6.2	4.6	9.15
The number of $(n,2)$ short-cuts	1978.6	5210.4	7113.8	8617.6	9827.4	14671.6	15787.6	18178.8	13.04
The number of $(n,2)$ breaks	11.4	10.5	10.2	10.6	11.8	11.8	12.4	15.4	8.6
The number of multiple short-cuts	1789	2198	2261.2	3477.2	5460.4	7091.4	7287.2	8692.4	5.75
The number of ephemeral short-cuts	517	1009.2	1230.2	1810	2241.8	2310.2	3022.4	3645.6	6.61

probability equations. In detail, $(n,1)$ short-cuts have something to do with the hop counts of active path, network area size, transmission range and relative speed; but $(n,1)$ breaks and $(n,2)$ breaks only relate to network area size, transmission range and relative speed.

The similar simulations were made for PCA. Tables 4 and 5 show the statistical data and the average error between theoretic data and statistical data (we omitted the theoretic data which were shown in Tables 2 and 3).

5. Conclusions and further work

Path compression techniques can compensate for the function of path optimization effectively which on-demand routing protocol can not provide. Path compression techniques are effective components of Ad Hoc on-demand routing protocol. However, there is no effective path compression model, and the research for path compression techniques in Ad Hoc on-demand routing is lack of basic theory. This paper proposed a dynamic model of path compression techniques which considered the mobility and scalability of Ad Hoc networks and could deduce the quantitative relationship between pivotal compression events and network parameters. The simulation results showed that our compression model could analyze and evaluate path compression algorithm effectively in mobile Ad Hoc network environment.

As a typical application for our compression model, we had proposed a stability module of path compression techniques (SMPC) [13] which calculated the real-time stable compression period probabilistically based on dynamic compression model. Specially, SMPC reduces the average number of ephemeral short-cuts and multiple short-cuts to 14% and 59% respectively compared to PCA.

In addition, further improvement of the compression model is necessary. For example, we can consider the number of neighboring nodes. In fact, neighboring nodes will not participate in neither $(n, 1)$ short-cuts nor $(n, 2)$ short-cuts. Therefore, we can evaluate the density of the network in advance, and the neighboring nodes can be excluded when calculating the compression probability.

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